the Sobolev spaces $W_\infty^{(n)}[a, b]$ and their periodic analogues $\widetilde{W}_\infty^{(n)}$, an exceptional role is played by the polynomial perfect splines of degree n, i.e., those functions whose nth derivatives take, on intervals, the values c and $-c$.

This research monograph considers certain generalizations of these two problems and their associated extremal functions. The function classes considered are $W^rH^\omega[a, b]$ and $\widetilde{W^rH^\omega}$ for certain sets of ω. These are defined in the following manner. Let $\omega(f; \cdot)$ denote the modulus of continuity of f, and let ω denote any fixed concave modulus of continuity. Then

$$W^rH^\omega[a, b] = \{f : f^{(r)} \in C[a, b],\ \omega(f^{(r)}; t) \leqslant \omega(t),\ t \in [0, b-a]\},$$

while

$$\widetilde{W^rH^\omega} = \{f\colon f \in W^rH^\omega(\mathbb{R}),\ f\ 2\pi \text{ periodic}\}.$$

For the case $\omega(t) = t$ we have $W^rH^\omega[a, b] = W_\infty^{(r+1)}[a, b]$ and $\widetilde{W^rH^\omega} = \widetilde{W}_\infty^{(r+1)}$. These function classes were introduced by S. M. Nikol'skii in the 1940s. They have been studied by N. P. Korneichuk and others.

To quote the author, the three main aims of this book are "(1) to introduce the notion and give the formulae for the perfect ω-splines in W^rH^ω; (2) to describe various extremal properties of perfect ω-splines by emphasizing the new phenomena and the old features inherited from polynomial perfect splines; and (3) to show examples of applications of the general theory of perfect splines in examples related to the computation of n-widths of classes $W^rH^\omega(I)$ and our solution of one of the most celebrated problems of real analysis—the Kolmogorov problem of sharp inequalities for intermediate derivatives in the Hölder classes $W^rH^\alpha(\mathbb{R}_+)$ and $W^rH^\alpha(\mathbb{R})$."

This book contains 17 chapters and 2 appendixes. Chapters 0–3, 6–9, and 14 contain the introduction, a review of known facts, and a proof of the main results concerning Chebyshev ω-splines. In Chapters 4, 5, and 10–12 and Appendixes A and B we find applications of these results to various forms of the Kolmogorov–Landau problem. Chapters 13, 15, and 16 contain solutions of the problem of n-widths for various such spaces.

This is a research monograph and not a textbook. As such it is detailed and not always easy to follow. However, it contains a wealth of information and is a must for any researcher in this field.

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Han-lin Chen, *Complex Harmonic Splines, Periodic Quasi-Wavelets. Theory and Applications*, Kluwer Academic, Dordrecht, 2000, xii + 226 pp.

The following review is taken from the preface of the book, with permission from the publisher.

This book, written by our distinguished colleague and friend, Professor Han-lin Chen of the Institute of Mathematics, Academia Sinica, Beijing, presents, for the first time in book form, his extensive work on complex harmonic splines with applications to wavelet analysis and the numerical solution of boundary integral equations. Professor Chen has worked in approximation theory and computational mathematics for over forty years. His scientific contributions are rich in variety and content. Through his publications and his many excellent Ph.D. students he has taken a leadership role in the development of these fields within China. This new book is yet another important addition to Professor Chen's quality research in computational mathematics.

In the last several decades, the theory of spline functions and their applications have greatly influenced numerous fields of applied mathematics, most notably, computational mathematics, wavelet analysis, and geometric modeling. Many books and monographs have been published studying real variable spline functions with a focus on their algebraic, analytic, and computational properties. In contrast, this book is the first to present the theory of complex harmonic spline functions and their relation to wavelet analysis with applications to the solution of partial differential equations and boundary integral equations of the second kind. The material presented in this book is unique and interesting. It provides a detailed summary of the important research results of the author and his group, and others in the field as well.

The book is organized into four chapters. Chapter I provides a rigorous study of the functional and geometrical properties of complex harmonic spline functions. Specifically, it contains the general theory of the interpolating and quasi-interpolating complex spline functions on the boundary of the unit disk. It also contains a discussion how the boundary values of complex harmonic spline functions influence their interior behavior. An algorithm for the computation of complex harmonic spline functions is also provided. In Chapter II various types of periodic quasi-wavelets are constructed using real and complex spline functions as generators. The orthogonality and least number of terms in the decomposition formulas for periodic quasi-wavelets, which are very important in applications, are thoroughly discussed. In Chapter III, the author applies periodic quasi-wavelets to solve boundary value problems for the two-dimensional Helmholtz equation by reducing it to a Fredholm integral equation of the second kind with a weakly singular kernel. Under certain smoothness conditions on the coefficients and the stiffness matrix being given, it is proved that the order of complexity of this algorithm is $O(N)$, where N represents the number of unknowns. In Chapter IV another type of periodic wavelets is constructed. These explicitly given wavelets possess the following important properties: interpolation, localization, symmetry, regularity up to any prescribed order, real-valued, and biorthogonal. Some illustrative examples are provided.

In summary, this book is a rigorous presentation of the numerous interesting mathematical properties and physical applications of complex harmonic spline functions, which is suitable not only as a reference source but also as a textbook for a special topics course or seminar. We are delighted to see the publication of this book and hope that it will foster new research and applications of complex harmonic splines and wavelets. We enthusiastically recommend it to the mathematics and engineering communities.

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Boris Osilenker, *Fourier Series in Orthogonal Polynomials*, World Scientific, Singapore, 1999, vi + 287 pp.

A classical topic in approximation theory is polynomial approximation. The best polynomial approximation of a function in a Hilbert space is obtained by taking the orthogonal projection of the function onto the linear space of polynomials of degree at most n. If one uses a basis of orthogonal functions, then this orthogonal projection becomes a partial sum of the Fourier series. Hence orthogonal polynomials and Fourier series in orthogonal polynomials are the building blocks for best polynomial approximation in a Hilbert space. The underlying theory is quite classical by now and is covered in the standard books on orthogonal polynomials (Szegő's *Orthogonal Polynomials* from 1939 and Freud's *Orthogonal Polynomials* from 1971, to name two of the most relevant books).

The book under review basically covers the standard material and adds some new material, mostly from the author's research results. Chapter 1 deals with some preliminaries, such as topics from function theory of a real variable, some topics from functional analysis (Banach–Steinhaus theorem), interpolation theorems (Riesz–Thorin and Marcinkiewicz), and